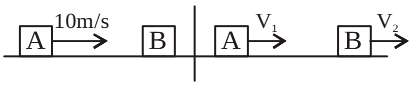
FINAL TEST SERIES XI JEE
TEST-04 ANSWER KEY

Test Date :09-02-2020

[PHYSICS]

1. (A) Bouyancy force acts at point ‘O’
The torque of bouyancy about the point C is clockwise

2. (C)



$m \times 10 = mv_1 + mv_1$

$\Rightarrow \quad 10 = v_1 + v_2 \qquad$ .....(i)

and $\frac{1}{2} \times 10 = v_2 - v_1 \qquad$ ....(ii)

From i and ii

$$v_1 = \frac{5}{2} m/s; v_2 = \frac{15}{2} m/s$$

Distance between the two blocks

$S = (-v_1 + v_2).t$

$$= \left(-\frac{5}{2} + \frac{15}{2}\right) \times 5 = 25m$$

3. (D) $F = ma = 5(x^2 - 3x + 2); F = 0 \Rightarrow \quad x = 1$ and 2

$\frac{dF}{dx} = 5(2x-3);$ at $x = 1 \quad \frac{dF}{dx} = -ve \Rightarrow$ stable equilibrium

and at t = 2 $\frac{dF}{dx} = +ve \Rightarrow$ unstable equilibrium

4. (A) For antinode $f_1 = \frac{(2n_1 - 1)}{4\ell}\sqrt{\frac{\ell}{\mu}}$

$$f_2 = \frac{(2n_2-1)}{4\ell}\sqrt{\frac{\ell}{9\mu}} = \frac{(2n_2-1)}{12\ell}\sqrt{\frac{\ell}{\mu}}$$

$f = \frac{1}{4\ell}\sqrt{\frac{\ell}{\mu}} \qquad \frac{(2n_1-1)}{4\ell} = \frac{2n_2 - 1}{12\ell}$

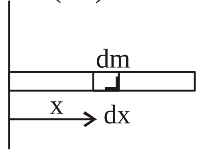
$6n_1 - 3 = 2n_2 - 1$

$6n_1 - 2n_2 = 2 \qquad n_1 = 1, n_2 = 2$

$3n_1 - n_2 = 1$

$dI = (dm)x^2$

5. (B)



$I_{net} = \int (dm)x^2$

$$I_{net} = \int_{x=0}^{x=2} (10x^2 dx)x^2$$

$I_{net} = 64$ unit

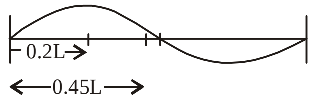
6. (B) Applying angular momentum conservation about the rotating axis

$L_i = L_f$

$$O + mvR = \left(\frac{mR^2}{2} + mR^2\right)\omega_f$$

$$\omega = \frac{2V}{3R}$$

7. (B) It is obvious that particle at 0.2 L will have larger amplitude that particle at 0.45 L, 0.5 L being the node and 0.25 being antinode



8. (A) Initially distance of suspension point from

$CM = \ell = \frac{1}{2}m$

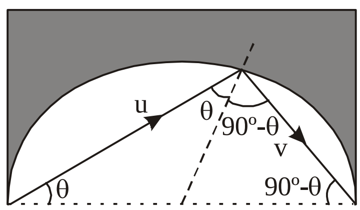
To get the same time period, we can hinge the rod at distance $\frac{K_{cm}^2}{\ell} = \frac{\left(\frac{1}{\sqrt{12}}\right)^2}{1/2} = \frac{1}{6}m$

Here $I_{cm} = \frac{mL^2}{12} = mK_{cm}^2 \Rightarrow K_{cm} = \frac{L}{\sqrt{12}} = \frac{1}{\sqrt{12}}$

9. (C) The coefficient of viscosity is the ratio of tangential stress on top surface of film (exerted by block) to that of velocity gradient (vertically downwards) of film. Since mass m moves with constant velocity, the string exerts a force equal to mg on plate towards right. Hence oil shall exert tangential force mg on plate towards left.

$$\therefore \quad \eta = \frac{F/A}{(v-0)\Delta x} = \frac{125 \times 1000/10 \times 20}{(5.0)/0.02} = 2.5 \text{dyne–s/cm}^2$$

10. (A)



$v \sin\theta = e\, u \cos\theta$

$v\cos\theta = u\sin\theta$

$\tan\theta = \dfrac{e}{\tan\theta}$

$e = \tan^2\theta = \dfrac{1}{3}$

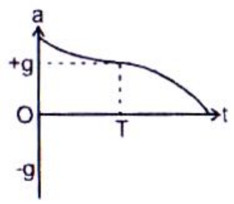
11. (B) $Mgx = \dfrac{1}{2}kx^2 + \dfrac{1}{2}I\omega^2 + \dfrac{1}{2}Mv^2$

$1\times10\times0.1 = \dfrac{1}{2}[50(0.1)^2 + \dfrac{1}{2}(0.2)\left(\dfrac{V}{0.2}\right)^2 + \dfrac{V^2}{2}]$

$\Rightarrow \quad V = \dfrac{1}{2}\,m/s$

12. (B) $\dfrac{25-0}{100} = \dfrac{P-25}{200} \quad \Rightarrow \quad P = 75^o$

13. (C) As the ball ascends upwards in addition to weight, air friction also acts downwards. Hence the initial magnitude of acceleration will be greater than magnitude of acceleration due to gravity. So the only possible option is



14. (C) $\dfrac{R'}{t'} = \dfrac{R(1+\alpha\Delta\theta)}{t(1+\alpha\Delta\theta)} = 1000$ .Hence the ratio $\dfrac{t}{R}$ will remain constant on heating

15 (B) $F_{net} = F_{dynamic} + F_{static}$

At any time t when x length is on the table then :

$F_{static} = x\lambda g$

$F_{dynamic} = v_{rel}\left(\dfrac{\lambda dx}{dt}\right) = \sqrt{2gx}\times\lambda\sqrt{2gx} = 2gx\lambda$

$F_{dynamic} = 3\times\left(\dfrac{1}{2}gt^2\right)\lambda g$

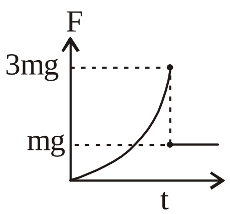
$\Rightarrow \quad F_{net} = 3\lambda g\, 1/2\, gt^2$

$= \dfrac{3}{2}\lambda g^2t^2 = \dfrac{3}{2}\dfrac{mg^2t^2}{\ell}$

At $t = \sqrt{2\ell/g}$ all the chain will be on table so

$F_{net} = 0 + mg$ (since at this position $F_{dyn} = 0$)

So the correct graph is represented by option (b)



16. (B) $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

$\Rightarrow \quad R = \dfrac{R_1R_2}{R_1+R_2} = \dfrac{3.0\times6.0}{9.0} = 2.0\Omega$

$\therefore \quad +\dfrac{\delta R}{R^2} = +\dfrac{\delta R_1}{R_1^2} + \dfrac{\delta R_2}{R_2^2}$

$= \dfrac{0.1}{(3)^2} + \dfrac{0.5}{(6)^2} = \dfrac{0.1}{4}$

Hence the correct choice is (b)

17. (B) In addition (or substraction),the last significant digit of sum (or difference) occupies the same relative position as the last significant digit of quantities being added (or substracted)

18. (A)

$\vec{v} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j} - gt\,\hat{j}$

$\vec{a} = -g\,\hat{j}$

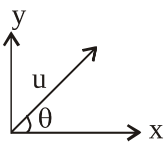
$\vec{a}\times\vec{v} = u\cos\theta g\,\hat{k} = \text{constant}$

At highest point

$\vec{a} = -g\,\hat{j}$

$\vec{v} = u\cos\theta\,\hat{i}$

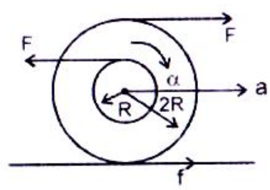
$\Rightarrow \quad \vec{a}.\vec{v} = 0$



19. (A)

From t = 0 to 5 S	$a = 2m/s^2$	$T = m(g+a) = 144N$
From t = 5 to 10 S	$a = 0$	$T = mg = 120\,N$
From t = 10 to 15 S	$a = 4m/s^2$	$T = m(g+a) = 168\,N$

20. (B)



$f = 2a$

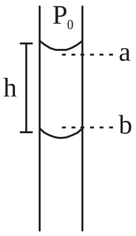
$FR - f\,2R = 4\dfrac{a}{2R} = 2a$

$F - 2f = f \quad \Rightarrow \quad f = \dfrac{F}{3} = \dfrac{10}{3}N$

21. (2) $E = \dfrac{1}{2}kA^2e^{-bt/m}$

$t = \dfrac{\ell n\,2}{b}m = \dfrac{\ell n2}{\ell n2}2 = 2\sec$

22. (6)



$P_a = P_0 - \frac{2T}{r}$

$P_b = P_0 + \frac{2T}{r}$

also, $P_b = P_a + dgh$

substuting values

$h = \frac{4T}{rdg} = \frac{4 \times 0.75}{\left(\frac{1}{2} \times 10^{-3}\right) \times \left(1 \times 10^{3}\right) \times 10} = 6 \times 10^{-2}\,m = 6cm$

23. (0) Loss in heat from calorimeter + water, as temperature changes from 10ºC to 0ºC

$= m_1C_1 10 + m_2C_2 10 \quad = 1 \times 1 \times 10 + 1 \times 0.1 \times 10 = 11$ kcal

Gain in heat of ice as its temperature changes from – 11ºC to 0ºC = $m_3C_3 \times 11 = 2 \times 0.5 \times 11 = 11$ kcal

Hence ice and water will coexist at 0ºC without any phase change

24. (6) $f' = (2n-1)\left(\frac{v}{4\ell}\right)$

and ℓ is length of pipe.

f′ = n × fundamental frequency

∴ We know that human ear can hear frequencies upto 20,000 Hz, hence

20,000 = n × 1500

$\Rightarrow \quad n = \frac{20000}{1500} \approx 13$

maximum possible harmonics obtained are

1, 3, 5, 7, 9, 11, 13

Hence, man can hear upto 13th harmonic

So number of overtones heard = 7 – 1 = 6

25. 5

[CHEMISTRY]

26. (A) $\frac{\text{Mass of sulphur}}{\text{Mol. mass of compound}} \times 100$ = % of sulphur

$\therefore \quad \left(\frac{2 \times 32}{M}\right) \times 100 = 0.032$

∴ M = 2,00,000

27. (D) Those reaction in which oxidation number of any element do not change not a redox reaction.

$AgCl + NH_3 \longrightarrow [Ag(NH_3)_2]Cl$.

28. (A) $1 \times 10^3\,kg/m^3 = 1$ g/mL. [Since, $1m^3 = 10^6\,cm^3 = 10^6$ mL].

= 1 gm/cc

6.022×10^{23} H_2O molecule weigh ———— 18 g

1 H_2O molecule weigh ————

$\frac{18}{6.022 \times 10^{23}}\,g = 3 \times 10^{-23}\,g$

$d = \frac{\text{mass}}{\text{volume}}$,

So, volume = $\frac{3 \times 10^{-23}\,g}{1(g/mL)} = 3 \times 10^{-23}$ mL.

29. (D) $PE = -\frac{KZe^2}{r}$.

30. (A) Energy of one photon = $\frac{12400}{6200}$ = 2 eV = 2 × 96

= 192 KJ mol^{-1}

∴ % of energy of photon converted to K.E. of A atoms

$= \frac{192 - 144}{192} \times 100 = \frac{48}{192} \times 100 = 25\%$

31. (A) Order of energy → Violet > Blue > yellow > red

Order of energy → $E_{2\to1} > E_{5\to2} > E_{6\to3} > E_{4\to3}$

∴ Violet (2 → 1), Blue (5 → 2), yellow (6 → 3), Red (4 → 3)

32. (D) Let, vol of containers be V & temp be T

$P_1 = 100$ mm $\quad P_2 = 400$mm

$\therefore\ n_1 = \frac{P_1V}{RT} \quad \& \quad n_2 = \frac{P_2V}{RT}$

$\therefore\ n_1 + n_2 = \frac{(P_1 + P_2) \times V}{RT}$

After joining two containers final vol = (V+V) = 2V(for gases)

$\therefore\ P_{final} = \frac{(n_1 + n_2)RT}{V_{final}} = \frac{(P_1 + P_2) \times V}{RT} \times \frac{RT}{2V}$

$= \frac{(P_1 + P_2)}{2}$

$= \frac{(100 + 400)mm}{2} = 250$ mm.

33. (D) Use formula $2\pi r_n = n\lambda$

We can’t apply Bohr radius formula for Be^{2+}

$2\pi r_n = n\lambda$

34. (A)

35. (D) $n_T = n_1 + n_2 + n_3 + \ldots\ldots\ldots\ldots\ldots\ldots$

$\frac{P_T.V_T}{RT} = \frac{P_1V_1}{RT} + \frac{P_2V_2}{RT} + \ldots\ldots\ldots\ldots\ldots\ldots = \Sigma P_iV_i$

$P_TV_T \quad = \Sigma P_iV_i$

$= 2PV + \frac{P.V}{2} + \frac{P}{2}.\frac{P}{4} + \frac{P}{4}.\frac{V}{8} + \ldots\ldots\ldots\ldots\ldots$

$= 2PV\,[\,1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$P_TV_T \quad = 2PV \frac{1}{1 - \frac{1}{4}} = 2PV.\frac{4}{3}$

$V_T \ = V_1 + V_2 + V_3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= V + \frac{V}{2} + \frac{V}{4} + \frac{V}{8} \ \ldots\ldots\ldots\ldots\ldots$

$= V\left[1 + \frac{1}{2} + \ldots\ldots\ldots\ldots\ldots\ldots\right] = V\frac{1}{1 - \frac{1}{2}} = 2V$

$\therefore\ P_T.\,2V = 2PV.\frac{4}{3}$

$P_T = \frac{4}{3}P$

36. (D)
37. (A) Factual
38. (D)
39. (A) $H_2(g) + Br_2(g) \rightleftharpoons 2HBr(g)$

Intial pressures 10.0 bar 0 0

At equilibrium (10.0-p) p/2 p/2

$$K_p = \frac{p^2_{HBr}}{p_{H_2} \times p_{Br_2}}$$

$$1.6 \times 10^5 = \frac{(10-p)^2}{(p/2)(p/2)}$$

Taking square root of both sides

$$4 \times 10^2 = \frac{10-p}{p/2}$$

200 p = 10 - p ;

$$p = \frac{10}{201} \text{ bar}$$

$$p_{H_2} = p/2 = \frac{1}{2}\left(\frac{10}{201}\right) \text{bar} = 2.5 \times 10^{-2} \text{bar} ;$$

$p_{Br_2} = p/2 = 2.5 \times 10^{-2}$ bar ; $p_{HBr} = 10 - p \approx 10$ bar .

40. (C)
41. (A,) $K_p = (p_{H_2O})^4 = 2.56 \times 10^{-10}$ atm^4

$\therefore$ $p_{H_2O} = 4 \times 10^{-3}$ atm $= 4 \times 10^{-3} \times 760 = 3.04$ torr.

Partial pressure of water vapour in air $= \frac{40}{100} \times 12.5 = 5$

So, the amount of water vapour in air should decrease to decrease value of partial pressure of water vapour from 5 torr to the equilibrium value (3.04 torr).

so, mass of $CuSO_4.5H_2O$ will increase and mass of $CuSO_4.H_2O$ will decrease.

42. (B) (a) $W = -nRT \ln \frac{V_2}{V_1}$

$W = -P_1V_1 \ln \frac{V_2}{V_1} = -14 \times 0.03 \ln \frac{0.06}{0.03}$ bar m^3 = $-14 \times 0.7 \times 0.03$ = **– 0.294 bar m³ Ans.**

43. (A) At A and D the temperatures of the gas will be equal, so $\Delta E = 0$, $\Delta H = 0$

Now $w = W_{AB} + W_{BC} + W_{CD} = -P_0V_0 - 2P_0V_0 \ln 2 + P_0V_0 = -2P_0V_0 \ln 2$

and $q = -W = 2P_0V_0 \ln 2$

44. (B) $CS_2(\ell) + 3O_2(g) \longrightarrow CO_2(g) + 2SO_2(g)$;

$\Delta H^o_{rxn.} = 5 \times -215 = -1075$ kJ

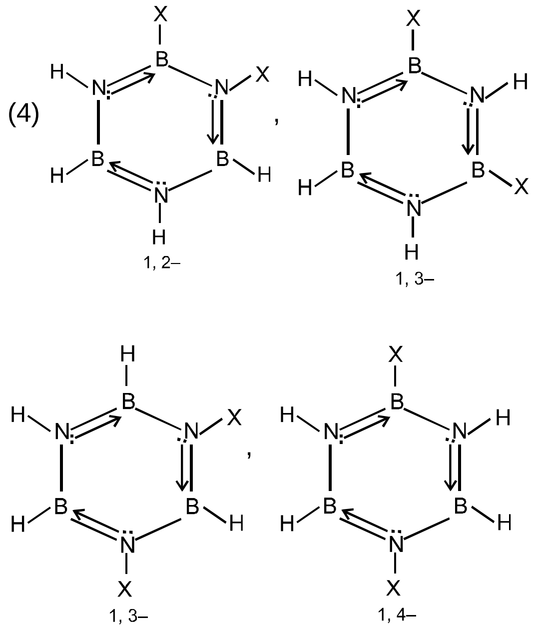
$\Delta H^o_{rxn.} = \Delta H^o_f(CO_2) + 2 \times \Delta H^o_f(SO_2) - \Delta H^o_f(CS_2)$

$\Delta H^o_{rxn.} = (-393.5 - 2 \times 296.8) - (-1075)$

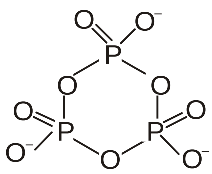
$\Delta H^o_{rxn.} = 87.9$

45. (D) $= \frac{3y - 4x}{3}$ kCal mol^{-1}.

46. (4)



47. (3) The cyclic methphosphate ion is



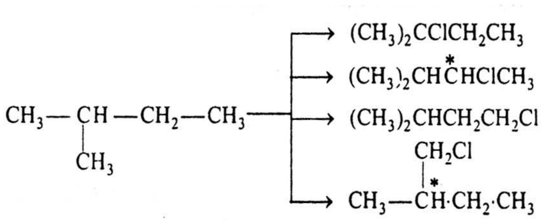
48.

$3Fe^o + 8e \longrightarrow (Fe^{+8/3})_3$

$[(H^+)_2 \longrightarrow (H^o)_2 + 2e] \times 4$

$3Fe + 4H_2O \longrightarrow Fe_3O_4 + 4H_2$

49.



50. $(CH_3)_2CH \cdot CH_2 \cdot CH{=}CH_2$

$(CH_3)_2CH \cdot CH{=}CHCH_3$

$(CH_3)_2C{=}CH \cdot CH_2 \cdot CH_3$

$CH_2{=}C(CH_3){-}CH_2 \cdot CH_2 \cdot CH_3$

[MATHEMATICS]

51. On rationalizing; we get ;

$$\frac{1-\sin x+1+\sin x+2|\cos x|}{1-\sin x-1-\sin x}=\frac{2(1+|\cos x|)}{-2(\sin x)}$$

$$=\frac{1-\cos x}{-(\sin x)}$$

52. Let $x = 3\cos\theta$; $y = 3\sin\theta$

$z = 2\cos\phi$; $t = 2\sin\phi$

$\therefore$ $6\cos\theta\cdot\sin\phi - 6\sin\theta\cos\phi = 6$

$6\sin(\theta-\phi) = 1$

$\sin(\phi-\theta) = 1$

$\phi = 90° + \theta$; $\phi - \theta = 90°$

$\therefore$ $x = 3\cos\theta$; $y = 3\sin\theta$

$z = -2\sin\theta$; $t = 2\cos\theta$

$p = xz = -6\sin\theta\cos\theta = -3\sin 2\theta$

$\therefore$ $p_{max} = 3$

53. $x^2+px+q=0$

Now, $\tan 30° + \tan 15° = -p$

and $\tan 30°\cdot\tan 15° = q$

$\therefore$ $\tan 45° = \tan(30° + 15°)$

$$=\frac{\tan 30°+\tan 15°}{1-\tan 30°\cdot\tan 15°}=\frac{-p}{1-q}=1$$

$\Rightarrow$ $q - p = 1$

Hence, $(2 + q - p) = 3$

54. $417 = 17 + (n-1)4$

$\Rightarrow$ $400 = 4(n-1)$

$\Rightarrow$ $n = 101$

Similarly, $466 = 16 + (m-1)5$

$\Rightarrow$ $450 = 5(m-1)$

$\Rightarrow$ $m = 91$

Let T_n is common to both for some n for which m is an integer.

$17 + (n-1)4 = 16 + (m-1)5$

$1 + 4n - 4 = 5m - 5$

$5m = 4n + 2$

55. $A=\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\frac{1}{5^4}+\frac{1}{6^4}$

$A=\left(\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots\right)+\frac{1}{2^4}\left(\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots\right)$

$A = B + \frac{1}{16}A$

$\frac{15}{16}A = B$

$\Rightarrow \frac{A}{B}=\frac{16}{15}$

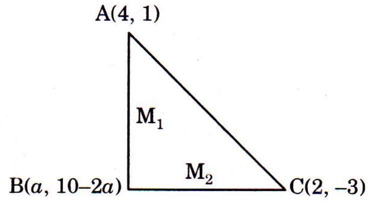
56. $\sin x + \sin 5x = \sin 2x + \sin 4x$

$2\sin 3x\cos 2x = 2\sin 3x\cos x$

$2\sin 3x[\cos 2x - \cos x] = 0$

On solving we get $x = \frac{n\pi}{3}$

57. $M_1M_2 = -1$



$\frac{9-2a}{a-4}\times\frac{13-2a}{a-2}=-1$

$117 - 26a - 18a + 4a^2 = -(a^2 - 6a + 8)$

$5a^2 - 50a + 125 = 0$

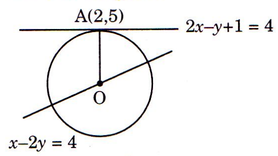
$a = 5$

So B is (5, 0)

So area $=\frac{1}{2}AB\times AC=\frac{1}{2}\sqrt{3}\times 3\sqrt{2}=3$

58. Compute perpendicular distance from (1, 0) to the radical axis of two circles.

59. $2x - y + 1 = 0$ is tangent.



Slope of line $OA = -\frac{1}{2}$

Equation of OA, $(y-5) = -\frac{1}{2}(x-2)$

$2y - 10 = -x + 2$

$x + 2y = 12$

$\therefore$ Intersection with $x - 2y = 4$ will give coordinates of centre.

Solving we get (8, 2).

Distance $OA=\sqrt{(8-2)^2+(2-5)^2}=\sqrt{36+9}$

$=\sqrt{45}=3\sqrt{5}$

60. Total – All four different $= 9\times 10^3 - 9\cdot 9\cdot 8\cdot 7 = 4464$

Or

Ans. 2204; all 4 digit even number – number of 4 digit even numbers with different digit.

61. ${}^{13}C_{10}$ – Number of ways in which he can reject 3 questions from the first five

Or

${}^{13}C_{10} - {}^{5}C_3 = 286 - 10 = 276$

Or

${}^{5}C_3\cdot{}^{8}C_7 + {}^{5}C_4\cdot{}^{8}C_6 + {}^{5}C_5\cdot{}^{8}C_5 = 276.$

[Note that ${}^{5}C_3\cdot{}^{10}C_7$ is wrong (cases repeat).

62. $2^n\left(1+\frac{x}{6}\right)^n$

$\Rightarrow \quad T_{r+1} = 2^n \cdot {}^nC_r\left(\frac{x}{6}\right)^r$

$\Rightarrow \quad 2^n \cdot {}^nC_7 \cdot \frac{1}{6^7} = 2^n \cdot {}^nC_8 \cdot \frac{1}{6^8}$

$\Rightarrow \quad 6 \cdot {}^nC_7 = {}^nC_8$

$\Rightarrow \quad n - 7 = 48$

$\Rightarrow \quad n = 55$

63. Let $(x^3-1)^{1/2} = y$

$\therefore \quad E = (x+y)^5 + (x-y)^5$

$= 2\,[{}^5C_0\,x^5 + {}^5C_2\,x^3y^2 + {}^5C_4\,xy^4]$

$E = 2\,[x^5 + 10x^3(x^3-1) + 5x(x^3-1)^2]$

Hence degree is 7.

64.

65. $\frac{a^2}{2} = \lim_{h\to 0}\frac{\sinh}{h}1;\ a = \sqrt{2}$

66. . Let $f(x) + g(x) = F(x)$

$f(x) - g(x) = G(x)$

Since $\lim_{x\to a} F(x)$ and $\lim_{x\to a} G(x)$ exists

Hence $\lim_{x\to a}\frac{F(x)+G(x)}{2}$ and $\lim_{x\to a}\frac{F(x)-G(x)}{2}$ must exist *i.e.*, $\lim_{x\to a} f(x)\cdot g(x)$ also exists.

67. . For $x > 20$, $f(x) = x - 2$; $g(x) = f(x) - 2 = x - 4$

68. . A circle and a parabola can meet at most in four points. Thus maximum number of common chords in ${}^4C_2$, *i.e.*, 6.

69. We have $2ae = 13\sqrt{2}$ = focal length ...(i)

$\because \quad 2a = 26$

$\Rightarrow \quad a = 13$ (By focus-directrix property)

$\therefore$ On putting $a = 13$ in equation (i), we get

$2(13)e = 13\sqrt{2}$

$\Rightarrow \quad e = \frac{1}{\sqrt{2}}$

70. $d^2 = 4a^2e^2 = 4(a^2+b^2) = 4$

$\Rightarrow \quad d = 2$

$\Rightarrow$ (a)

71. $z = 0;\quad z = \pm 1;\quad z = \pm i;\quad z^3 = \bar{z}$

$\Rightarrow \quad |z|^3 = |\bar{z}| = |z|$

Note that $z^n = |\bar{z}|$ has $n+2$ solutions

Hence, $\quad |z| = 0$ or $|z|^2 = 1$

Again $\quad z^4 = z\bar{z} = |z|^2 = 1$

$\Rightarrow \quad z^4 = 1$

$\Rightarrow$ Total number of roots are 5

Note that the equation $z^n = \bar{z}$ will have $(n+2)$ solutions.

72. 7

73. $y = \frac{x^2+x+7}{x+2}$

$\Rightarrow \quad x^2 + x(1-y) + 7 - 2y = 0$

$\Rightarrow \quad D \geq 0$

$(1-y)^2 - 4(7-2y) \geq 0$

$1 + y^2 - 2y - 28 + 8y \geq 0$

$y^2 + 6y - 27 \geq 0$

$(y+9)(y-3) \geq 0$

$\Rightarrow$ Minimum positive integral value is 3.

74. Here, we have

$\cos 2x + c\sin x = 2c - 7$

$\Rightarrow \quad (1 - 2\sin^2 x) + c\sin x = 2c - 7$

$\Rightarrow \quad 2\sin^2 x - c\sin x + 2c - 8 = 0$

$\Rightarrow \quad \sin x = \frac{c \pm \sqrt{c^2 - 8(2c-8)}}{4}$

$\Rightarrow \quad \sin x = \frac{c-4}{2}$ or 2

But $\sin x = 2$ (Reject)

So, $-1 \leq \frac{c-4}{2} \leq 1$

$\Rightarrow \quad -2 \leq c - 4 \leq 2$

$\Rightarrow \quad 2 \leq c \leq 6$

$\therefore \quad c = 2, 3, 4, 5, 6.$

75. Equation of any circle through the points of intersection of given circles is :

$(x^2+y^2-6x+2y+4) + \lambda\,(x^2+y^2+2x-4y-6) = 0$

$\Rightarrow \quad x^2(1+\lambda) + y^2(1+\lambda) - 2x(3-\lambda) + 2y(1-2\lambda) + (4-6\lambda) = 0$

or $\quad x^2 + y^2 - \frac{2x(3-\lambda)}{(1+\lambda)} + \frac{2y(1-\lambda)}{(1+\lambda)} + \frac{(4-6\lambda)}{(1+\lambda)} = 0$ ...(i)

Its centre $\left\{\frac{3-\lambda}{1+\lambda}, \frac{2\lambda-1}{1+\lambda}\right\}$ lies on the line $y = x$

then $\quad \frac{2\lambda-1}{1+\lambda} = \frac{3-\lambda}{1+\lambda} \Rightarrow \lambda \neq 1$

$\therefore \quad 2\lambda - 1 = 3 - \lambda$

or $\quad 3\lambda = 4$

$\therefore \quad \lambda = \frac{4}{3}$

$\therefore$ Substituting the value of $\lambda = \frac{4}{3}$ in eq. (i), we get the required equation is $7x^2 + 7y^2 - 10x - 10y - 12 = 0$